The mechanism-based approach for age-period-cohort analysis

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Age-period-cohort models

- Age-period-cohort models have a long history in epidemiology, social science and econometrics
  - Fannon and Nielsen (2019), Fosse and Winship (2019), and Murphy and Yang (2018)
- The purpose is to assess how an outcome of an individual is ‘related’ to three different time variables:
  - the time since the individual was born (age)
  - the calendar date at which the outcome is observed (period)
  - the calendar date at which the individual was born (cohort)

A determinisic relation

- The three time variables are deterministically related:

$$\text{age} = \text{period} - \text{cohort}$$

  - e.g., an individual who is born in year 1950 is 20 years old in 1970:

$$20 = 1970 - 1950$$

Statistical associations

- The deterministic age-period-cohort relation is not problematic if we only care about statistical associations
- We can then:
  - model how the outcome is associated with, say, cohort and period, and
  - determine the outcome's association with age from fitted model by converting either cohort or period to age
- Usual goodness-of-fit tests relevant here as well...

Causal effects

- The typical aim is more ambitious: we want to estimate the ‘independent’ causal effects of age, period and cohort
  - rarely stated explicitly though
- An identifiability problem: cannot contrast different values of one time variable while holding the two others fixed
  - somewhat similar to attempting to adjust for a confounder that is perfectly correlated with the exposure

The constrained effects approach

- By far most common, traditionally
- Imposes parametric constraints on the three effects to make them identifiable
  - Firebaugh and Davis (1988), Glenn (1994), and Myers and Lee (1998): assume one of the effects is 0
  - more elaborate proposals as well Knoke and Hout (1974), Mason et al. (1973), and Nakamura (1986)
- Problems:
  - the constraints are often artificial; no *a priori* reason to believe that they hold
  - results are typically sensitive to the choice of constraints

The mechanism-based approach

- Utilizes mediators on causal pathways between age-period-cohort and the outcome
- A sufficiently informative set of mediators may ‘explain’ the age, period and cohort effects and make them identifiable
  - Heckman and Robb (1985), O’Brien (2000), and Winship and Harding (2008): mainly informal arguments, and no explicit causal estimand
  - Bijlsma et al. (2017) and Sjölander and Gabriel (2025): formal development with modern causal inference methods

Outline

Motivating example (Winship & Harding, 2008)

Nonparametric identification

Parametric estimation

Motivating example, revisited

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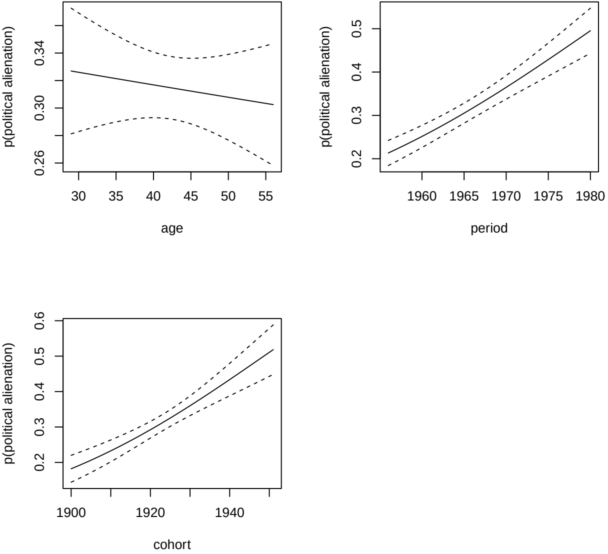
Motivating example, revisited

Data and aim

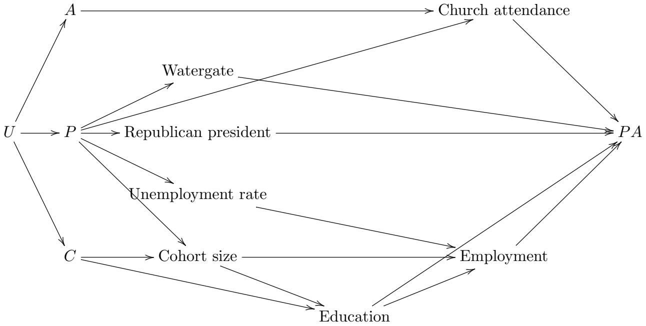
- Data from the American National Election Studies (ANES)
  - academically run national surveys of voters in the US, conducted around every presidential election
  - publicly available at https://electionstudies.org/
- Inclusion criterion: married white males age 29 to 56 surveyed in 1956, 1960, 1964, 1968, 1976, and 1980
  - $n = 1605$ (questionnaires, not individuals)
- Outcome: political alienation
  - binary indicator of the respondent agreeing with the statement '*I don't think officials care much what people like me think*'.

Statistical associations: one-by-one

$$\text{logit}\{p(\text{political alienation}|X)\} = \beta_0 + \beta_1 X,\ X \in \{\text{age, period, cohort}\}$$



Assumed causal diagram (Winship & Harding, 2008)



- The role of U is to enforce the relation $A = P - C$
- Key assumptions
  - **no unmeasured confounding**: no common causes of (A, P, C) and other variables
  - **partial exclusion**: no variable (mediator or outcome) is directly caused by both A, P and C – will modify this definition later to cover more general scenarios

Analysis (Winship & Harding, 2008)

- Regression of each variable (mediators and outcome) on its direct causes
  - no identifiability problem when fitting models due to partial exclusion assumption
- Path-specific effects through product-of-coefficient method
  - standard in linear structural equation modeling (Bollen, 1989)
  - presumed causal interpretation of estimated effects due to no unmeasured confounding assumption

Results (Winship & Harding, 2008)

	Estimate	95 Percent CI
Period effect (1976 vs. 1960)		
P → Watergate → PA	.4939	.3337, .6629
P → Republican president → PA	.3576	.2010, .5205
P → unemployment rate → employment → PA	.0021	−.0017, .0067
P → cohort size → employment → PA	.0018	−.0015, .0059
P → cohort size → education → PA	−.0099	−.0238, .0043
P → cohort size → education → employment → PA	−.0001	−.0003, .0001
P → church attendance → PA	.0487	.0163, .0973
Total	.8940	.7252, 1.0633
Cohort effect (1936-1939 vs. 1908-1911)		
C → cohort size → employment → PA	.0018	−.0015, .0059
C → cohort size → education → PA	−.0099	−.0238, .0043
C → cohort size → education → employment → PA	−.0001	−.0003, .0001
C → education → PA	−.1470	−.2321, −.0676
C → education → employment → PA	−.0012	−.0044, .0009
Total	−.1565	−.2446, −.0791
Age effect (37-40 vs. 49-52)		
A → church attendance → PA	.0077	−.0157, .0310
Total	.0077	−.0157, .0310
Grand total	.7453	.5590, .9214

Hmm...

- No causal estimand or proofs
  - not clear what is being estimated, i.e., how to interpret the obtained estimates
- Product-of-coefficient method is only valid for linear models
  - Winship and Harding (2008) used logistic and probit models
- All measured mediators used in the analysis
  - requires extensive regression modeling, which makes the analysis vulnerable to model misspecification bias

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Motivating example, revisited

A simple example

$$
\begin{tikzcd}
 & A \arrow[r] & M \arrow[dr] & \\
U \arrow[ur] \arrow[r] \arrow[dr] & P \arrow[rr] & & Y \\
 & C \arrow[urr] & &
\end{tikzcd}
$$

- The role of U is to enforce the relation $A = P - C$
- Key assumptions
  - **no unmeasured confounding**: no common causes of (A, P, C) and (M, Y)
  - **partial exclusion**:

    $$M \perp_d \{P, C\} | A$$

    $$Y \perp_d A | \{P, C, M\}$$

    where $V_1 \perp_d V_2 | V_3$ denotes V_1 and V_2 d-separated by conditioning on V_3

Causal estimand (Bijlsma et al., 2017)

- Let $Y(a, p, c)$ be the potential outcome for a given individual, if age, period and cohort were set to a, p and c
  - not necessarily obeying the deterministic relation $a = p - c$
- Let $E[Y(a, p, c)]$ be the mean potential outcome, if age, period and cohort were set to a, p and c for all individuals
- Causal effect of taking A from a to a', while holding (P, C) fixed at (p, c):

$$E[Y(a', p, c)] - E[Y(a, p, c)]$$

Quite controversial!

- An intervention that sets A, P, and C to fixed values is highly hypothetical
  - e.g., we have no time machines
- Is this a problem? A longstanding debate in causal inference!
  - Hernán (2005) and Holland (1986): one should only consider a counterfactual as meaningful if one can specify a practically feasible intervention that would make the counterfactual observable
  - Pearl (2018): '*counterfactuals and causal effects are defined independently of those [practically feasible] interventions and therefore, are not to be denied existence, or rendered "inconsistent" by the latter's imperfections*'; similar views in Glymour and Glymour, 2014
- What is the alternative?

Nonparametric identification (Sjölander & Gabriel, 2025)

$$
\begin{tikzcd}
 & A \arrow[r] & M \arrow[dr] & \\
U \arrow[ur] \arrow[r] \arrow[dr] & P \arrow[rr] & & Y \\
 & C \arrow[urr] & &
\end{tikzcd}
$$

$$
\begin{aligned}
E[Y(a,p,c)] &\overset{(1)}{=} E^*(Y|A=a,P=p,C=c) \\
&= \sum_m \Big\{ E^*(Y|A=a,P=p,C=c,M=m) \\
&\qquad\qquad \times p^*(M=m|A=a,P=p,C=c) \Big\} \\
&\overset{(2)}{=} \sum_m \underbrace{E(Y|P=p,C=c,M=m)p(M=m|A=a)}_{\text{identifiable from } p(A,P,C,M,Y)}
\end{aligned}
$$

- $E^*(\cdot)$, $p^*(\cdot)$: hypothetical world where (A, P, C) vary freely
- (1): no unmeasured confounding
- (2): partial exclusion

General result: data and assumptions

- Data: $A, P, C, \boldsymbol{M}_K = (M_1, ..., M_K), Y$
- Assumptions:
  - **no unmeasured confounding**: no common causes of (A, P, C) and $(\boldsymbol{M}_K, Y)$
  - **partial exclusion**: there exists proper subsets $\{R_1(A, P, C), ..., R_K(A, P, C)\}$ and $R_Y(A, P, C)$, with complements $\{R_1'(A, P, C), ..., R_K'(A, P, C)\}$ and $R_Y'(A, P, C)$, such that

    $$M_k \perp_d R_k'(A, P, C) | \{R_k(A, P, C), \boldsymbol{M}_{k-1}\} \text{ for } k = 1, ..., K$$

    and

    $$Y \perp_d R_Y'(A, P, C) | \{R_Y(A, P, C), \boldsymbol{M}_K\}$$

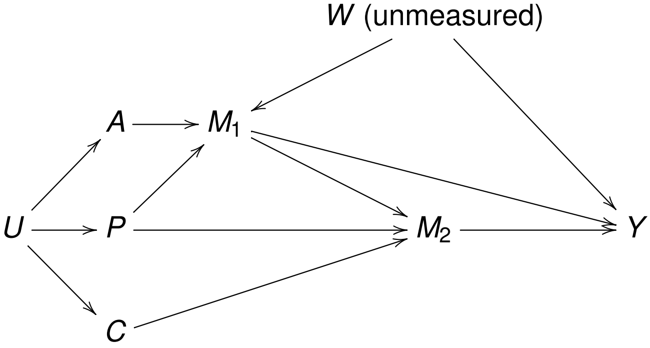
General result: the APC-formula

- Under the no unmeasured confounding and partial exclusion assumptions, $E[Y(a,p,c)]$ is identified as

$$\begin{aligned} E[Y(a,p,c)] &= \sum_{\boldsymbol{m}_K} \Big\{ E[Y \mid R_Y(a,p,c), \boldsymbol{M}_K = \boldsymbol{m}_K] \\ &\times \prod_{k=1}^{K} p[M_k = m_k \mid R_k(a,p,c), \boldsymbol{M}_{k-1} = \boldsymbol{m}_{k-1}] \Big\} \end{aligned}$$

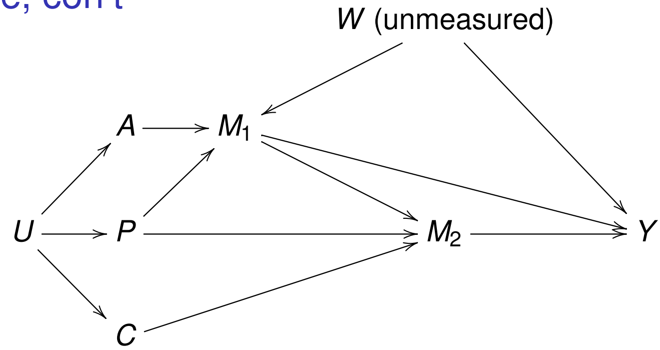
- Each term contains at most two of (a,p,c), so there is no identification problem
- Similar to the nonparametric G-formula in longitudinal studies with time-varying exposures and confounders
  - Robins (1986)

Example: identification with two mediators and unmeasured mediator-outcome confounding



- Cannot use M_1 alone for identification, since Y is not d-separated from any of (A, P, C) by conditioning on M_1
- Cannot use M_2 alone for identification, since M_2 is not d-separated from any of (A, P, C)

Example, con't

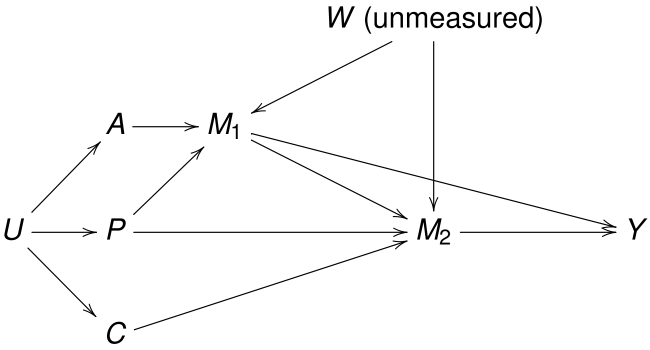


- Can use M_1 and M_2 together for identification since

$$M_1 \perp_d C$$
$$M_2 \perp_d A | \{P, C, M_1\}$$
$$Y \perp_d C | \{A, P, M_1, M_2\}$$

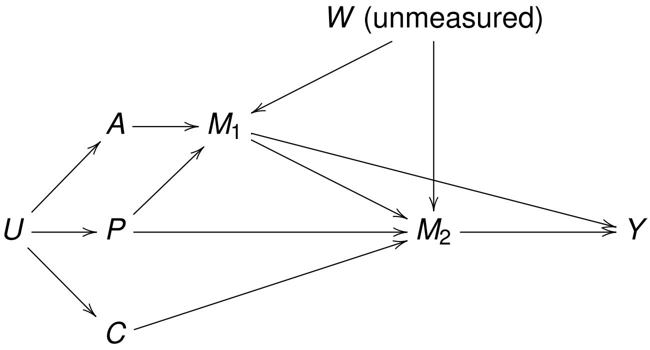
$$\begin{aligned} E[Y(a,p,c)] \quad = \quad & \sum_{m_1, m_2} \Big\{ E(Y | \underbrace{A = a, P = p}_{=R_Y(a,p,c)}, M_1 = m_1, M_2 = m_2) \\ \times \quad & p(M_2 = m_2 | \underbrace{P = p, C = c}_{=R_2(a,p,c)}, M_1 = m_1) p(M_1 = m_1 | \underbrace{A = a, P = p}_{R_1(a,p,c)}) \Big\}. \end{aligned}$$

Example: non-identifiability due to mediator-mediator confounding



- Cannot use M_1 alone for identification, since Y is not d-separated from any of (A, P, C) by conditioning on M_1
- Cannot use M_2 alone for identification, since M_2 is not d-separated from any of (A, P, C)

Example: non-identifiability due to mediator-mediator confounding, con't



- Cannot use M_1 and M_2 together for identification, since
  - M_2 is not d-separated from any of (A, P, C) by conditioning on M_1
  - M_1 is not d-separated from any of (A, P, C) by conditioning on M_2

Outline

Motivating example (Winship & Harding, 2008)

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Parametric estimation

Motivating example, revisited

Parametric models

$$p[M_k = m_k | R_k(a,p,c), \boldsymbol{M}_{k-1} = \boldsymbol{m}_{k-1}; \alpha_k] \quad \text{for } k = 1, ..., K$$
$$E[Y | R_Y(a,p,c), \boldsymbol{M}_K = \boldsymbol{m}_K; \beta]$$

$$\begin{aligned} E[Y(a,p,c)] &= \sum_{\boldsymbol{m}_K} \Big\{ E[Y | R_Y(a,p,c), \boldsymbol{M}_K = \boldsymbol{m}_K; \widehat{\beta}] \\ &\times \prod_{k=1}^{K} p[M_k = m_k | R_k(a,p,c), \boldsymbol{M}_{k-1} = \boldsymbol{m}_{k-1}; \widehat{\alpha}_k] \Big\} \end{aligned}$$

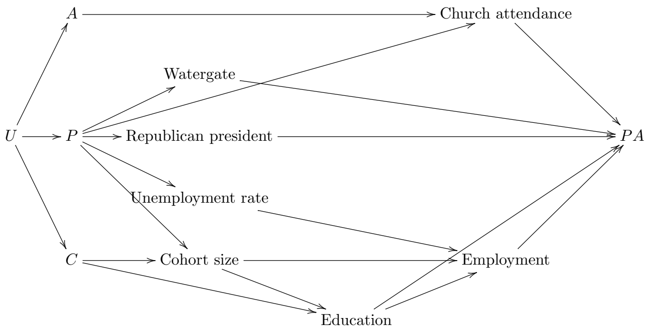
- Similar to parametric G-formula estimator of causal effects in longitudinal studies with time-varying confounding
  - Taubman et al. (2009) and Westreich et al. (2012)
- Low-dimensional $\boldsymbol{M}$: direct summation
- High-dimensional $\boldsymbol{M}$: Monte Carlo simulation

Monte Carlo simulation

1. Fit models to obtain estimates $\widehat{\alpha}_1, ..., \widehat{\alpha}_K, \widehat{\beta}$, and let N be large integer, e.g., $N = 10.000$.
2. For $i = 1, ..., N$, repeat steps 3-5.
3. Define
$$\widehat{m}_0^i(a, p, c) = \emptyset$$
4. For $k = 1, ..., K$, define
$$\widehat{\boldsymbol{m}}_{k-1}^i(a, p, c) = \{\widehat{m}_0^i(a, p, c), ..., \widehat{m}_{k-1}^i(a, p, c)\}$$
and generate a prediction $\widehat{m}_k^i(a, p, c)$ as a random draw from the fitted model
$$p[M_k = m_k | R_k(a, p, c), \boldsymbol{M}_{k-1} = \widehat{\boldsymbol{m}}_{k-1}^i(a, p, c); \widehat{\alpha}_k]$$
5. Define the prediction
$$\widehat{y}^i(a, p, c) = E[Y | R_Y(a, p, c), \boldsymbol{M}_K = \widehat{\boldsymbol{m}}_K^i(a, p, c); \widehat{\beta}]$$
6. Estimate $E[Y(a, p, c)]$ as
$$\widehat{E}[Y(a, p, c)] = \sum_{i=1}^{N} \widehat{y}^i(a, p, c)/N$$

Outline

Nonparametric identification



- It suffices to use church attendance for identification, since

$$\text{church attendance} \perp_d C | \{A, P\}$$

$$PA \perp_d A | \{P, C, \text{church attendance}\}$$

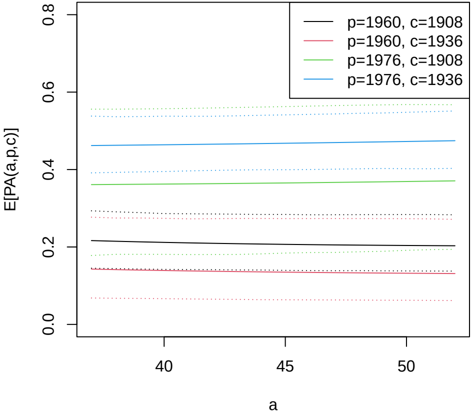
$$\begin{aligned} E[PA(a,p,c)] &= \sum_m \Big\{ E(PA | P = p, C = c, \text{church attendance} = m) \\ &\times \quad p(\text{church attendance} = m | A = a, P = p) \Big\}. \end{aligned}$$

Parametric estimation

$$
\begin{aligned}
E[PA(a,p,c)] &= \sum_m \Big\{ E(PA \mid P = p, C = c, \text{church attendance} = m) \\
&\times \quad p(\text{church attendance} = m \mid A = a, P = p) \Big\}.
\end{aligned}
$$

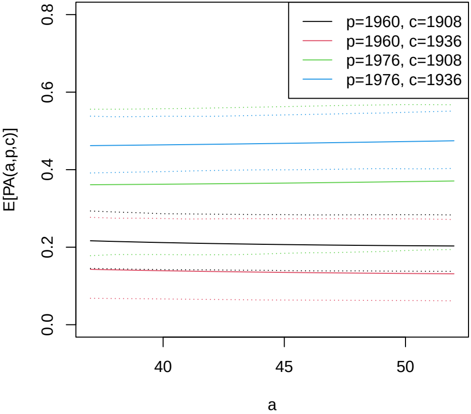
- Age: continuous (29-56)
- Period: categorical (1956, 1960, 1964, 1968, 1976, 1980)
- Cohort: continuous (1900-1951)
- Church attendance: categorical (never, seldom, regularly, often); multinomial logistic regression
- Political alienation: binary; ordinary (binomial) logistic regression
- Main effects and all two-way interactions in both regression models
- Direct summation over *m*, 95% confidence intervals with a nonparametric bootstrap, 1000 resamples

Results



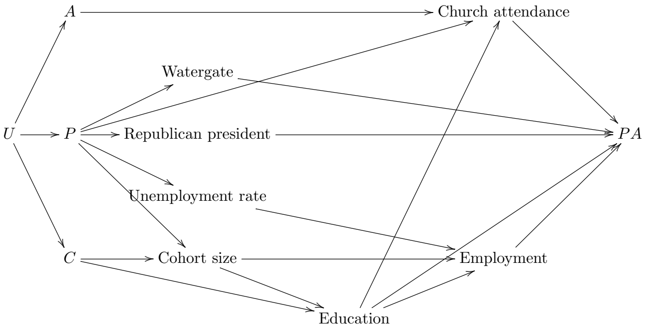
- Virtually no age effect
- Strong positive period effect: $E[PA(a, 1976, c)] > E[PA(a, 1960, c)]$ for all $\{a, c\}$
- Qualitatively similar to Winship and Harding (2008)

Results



- Strong cohort effect
  - negative when $p = 1960$:
    $E[PA(a, 1960, 1936)] < E[PA(a, 1960, 1908)]$ for all a
  - positive when $p = 1976$:
    $E[PA(a, 1976, 1936)] < E[PA(a, 1976, 1908)]$ for all a
  - unnoticed by Winship and Harding (2008) since they had no period-cohort interactions

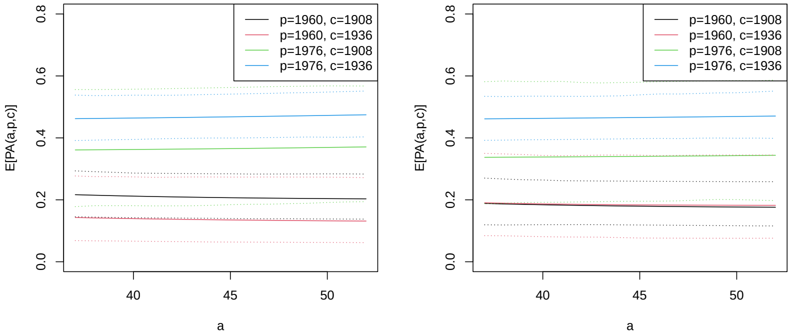
Nonparametric identification, cont'd



$$\text{education} \perp_d A | \{P, C\}$$
$$\text{church attendance} \perp_d C | \{A, P, \text{education}\}$$
$$PA \perp_d A | \{P, C, \text{education, church attendance}\}$$

$$
\begin{aligned}
& E[PA(a, p, c)] \\
& \quad = \sum_{m_1, m_2} \Big\{ E(PA | P = p, C = c, \text{education} = m_1, \text{church attendance} = m_2) \\
& \quad \times p(\text{church attendance} = m_2 | A = a, P = p, \text{education}) \\
& \quad \times p(\text{education} = m_1 | P = p, C = c) \Big\}
\end{aligned}
$$

Results



- No cohort effect when $p = 1960$

Summary

- If we only care about statistical associations, then the deterministic age-period-cohort relation is not a problem
- If we care about causal effects, then it poses both conceptual and identifiability problems
- Under no unmeasured confounding and partial exclusion, nonparametric identification is given by the APC-formula
- Estimation can be carried out with parametric models, similar to parametric G-formula estimation

Future work

- Less controversial causal estimand, without interventions on age-period-cohort?
- How to find the minimal sufficient set of mediators for identification?
  - work with Chihao Yan (student at biostat master program)
- Parametric model for $E[Y(a, p, c)]$, estimation with IPW?
  - work with Patrick Schnell, visiting associated professor from Ohio State University

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