

# Propensity score and measured confounding

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**Thank you for scholarship ☺**

# **Causal inference in epidemiology: recent methodological development**

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**Organizers:**

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# **AIMS of course:**

- a) Consider sources of error and uncertainty other than sampling error**
- b) Discuss recent methodological developments and how they relate to each other**
- c) Gain some practical experience of all these methods**

# Course contents

**Monday – Causal diagrams; Structural Equation Models; Confounding; Unmeasured confounding (instrumental variables);**

**Tuesday – Missing data; Multiple imputation; Sensitivity analysis;**

**Wednesday – Measured and unmeasured confounding; Propensity score; Inverse probability of treatment weighing**

**Thursday – Covariate Measurement error (instrument error, recall bias) Internal validation**

**Friday – Time varying exposure/confounding and mediation**

# Aim

- **Background**
- **Propensity score**
- **Applications of Propensity score**
- **An Example**

# Randomized trials

- Randomized trials (RT)
  - » Randomization to treatment or control
  - » Unbiased treatment effect is assumed

Average treatment effect =  $E(Y_1) - E(Y_0)$

( $E(\cdot)$ =expectation,  $Y_1$ =outcome in treated,  $Y_0$  = outcome in control)

»but RT **not** always possible (costs, morality)

# Observational studies (OS)

- Observational studies
  - » Inexpensive, Ethical, Feasible
- But limited by
  - » Not-randomized & selection bias\*, different comparision groups ("apples to oranges")
  - » High dimensional confounders with different values

\*A distortion resulting from the methods used to selcet study participants(Rothman KJ (2008) )

# Inference in Observational studies

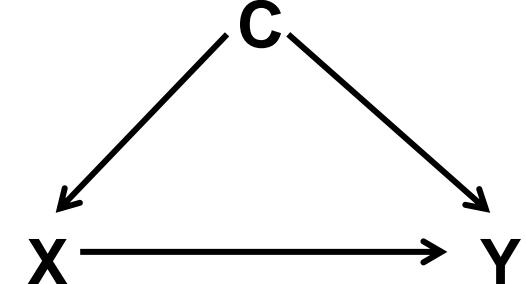
important information & important to develop better statistical analysis methods

To overcome limitations

- Strict inclusion & exclusion criterion
- Matching individuals in treated & control group
- Statistical techniques
  - » Regression, Stratification, Propensity score

# Effect estimation by regression

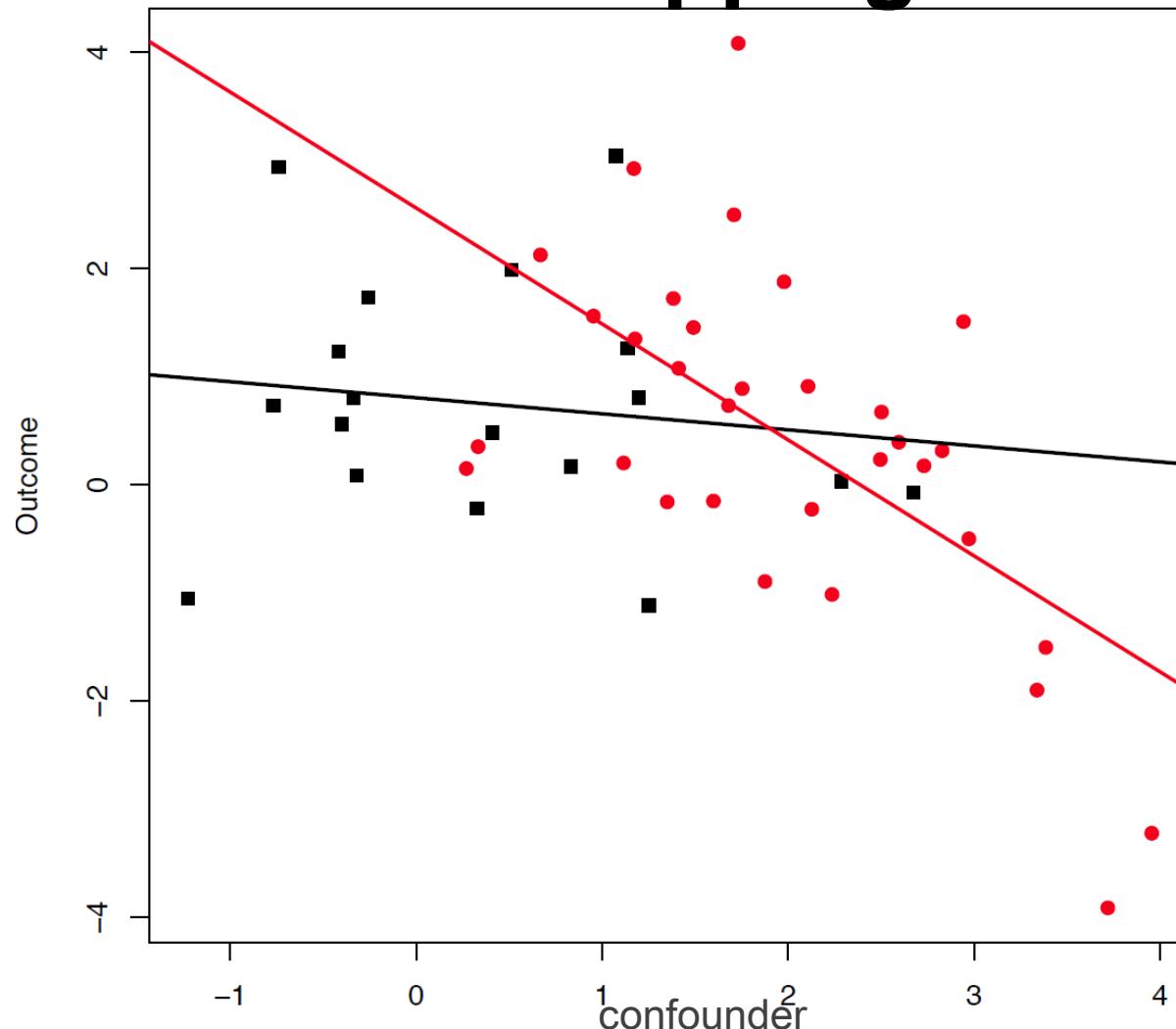
- $C$  = confounders
- $X$  = treatment (0,1)
- $Y$  = outcome



$$E(Y | X, C) = \alpha_0 + \alpha_1 C + \beta X$$

- The regression model compares individuals with treatment (0/1) & adjust for  $C$
- When little overlap in treated & control group, **extrapolation**
- Well fitted models may also produce **biased** treatment effect estimate

# Insufficient overlapping



# To overcome extrapolation

- Modelling association between exposure (X) & confounder (C) may have benefits

**Propensity score (PS)** (Rosenbaum & Rubin, Biometrika 1983)

The conditional probability of an individual getting treatment given her/his observed covariates (C)

$$PS = \text{pr}(X|C) = \left( \frac{\exp(\beta_0 + \sum \beta_i C)}{1 + \exp(\beta_0 + \sum \beta_i C)} \right)$$

\*All individuals have non-zero probability of receiving each treatment

# Propensity score

- Propensity score summarizes confounders
- Individuals with same propensity score equally likely to have been treated thus comparable
  - » Unless important cofounders are not measured
  - » Or PS was not modeled correctly

# Using PS

1. Stratifying (subclassifying) on PS

2. Matching on PS;

3. Adjusting for PS in linear models

$$E(Y | X, p(C)) = \alpha_0 + \alpha_1 p(C) + \beta X$$

4. Adjusting for PS & confounders in linear models

$$E(Y | X, p(C)) = \alpha_0 + \alpha_1 p(C) + \alpha_2 C + \beta X$$

increases precision & doubly robust, if PS or regression model is correct (Robins, Mark & Newey, 1992)

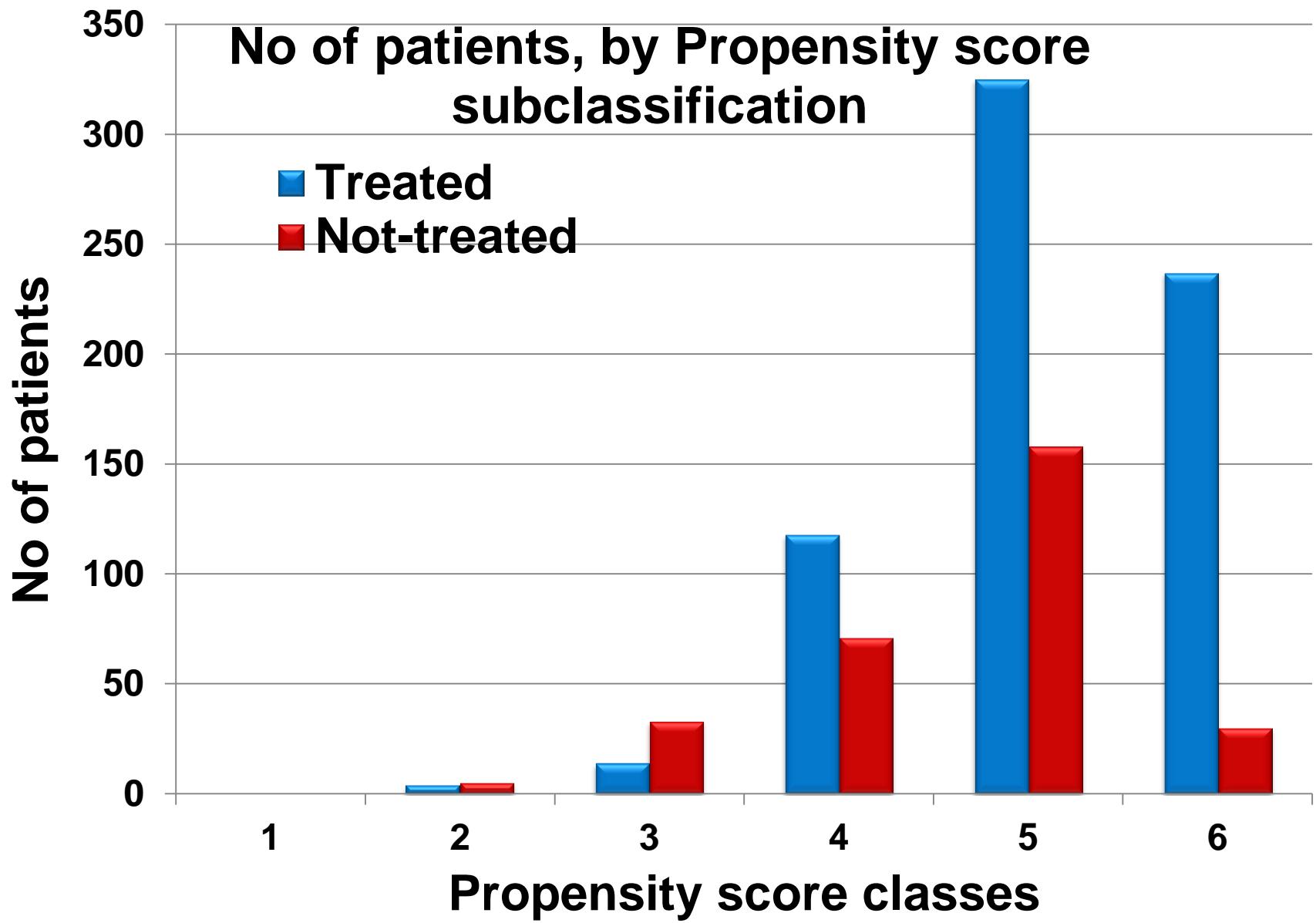
# Propensity score

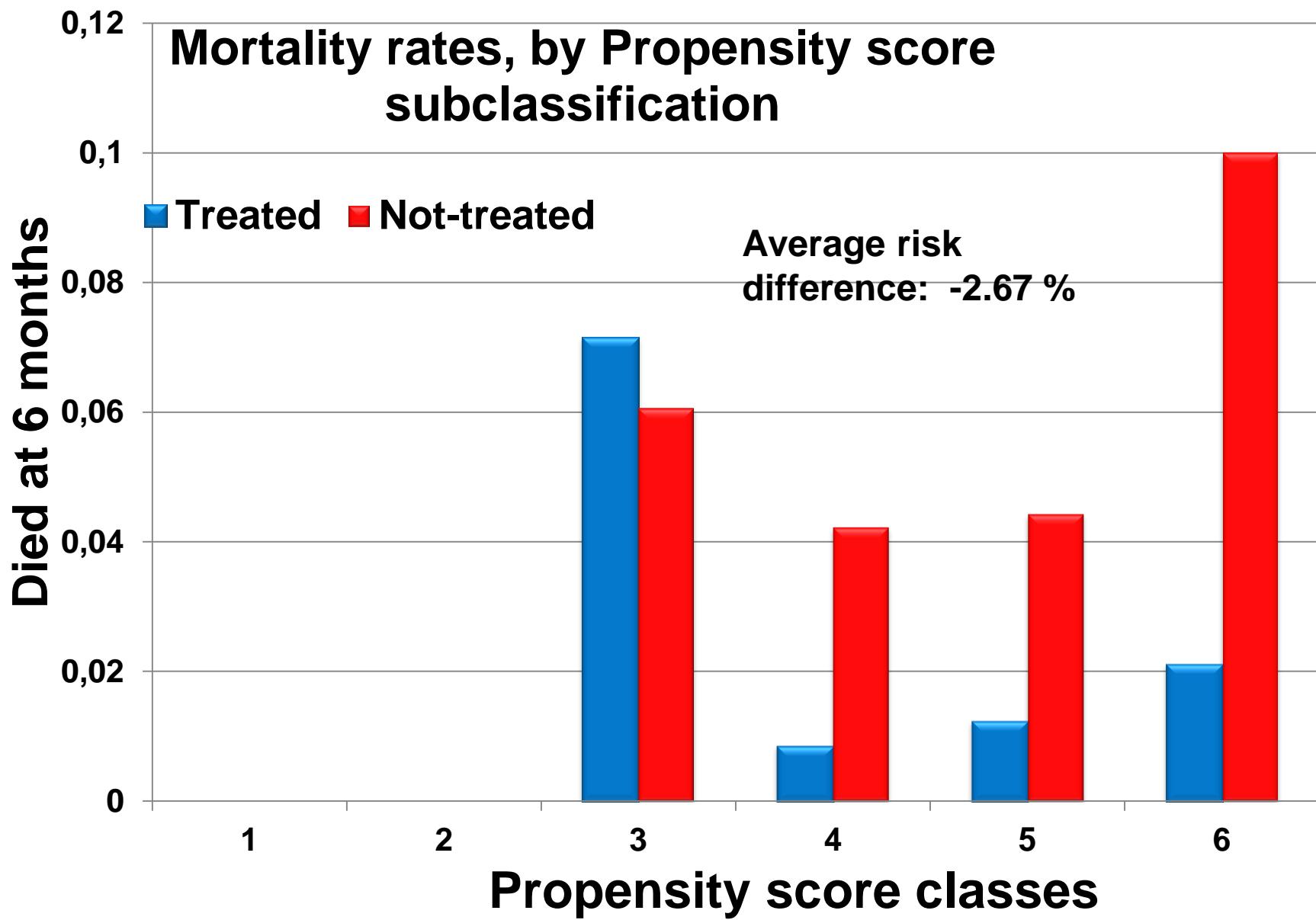
- Individuals in same strata have similar characteristic
- Outcomes are compared in each strata
- Mean or weighted mean yields overall treatment effect
- Matching by PS, treated & control groups more balanced on covariates

# An example

1. **N** = 996 patients at Ohio Heart Health in 1997
2. **Treatment**,  $X_1$  = Augmented Percutaneous Coronary Intervention (A-PCI) and  $X_0$  = only PCI
3. **Outcome**,  $Y$  = survival at least 6 months after treatment
4. **Confounders**,  $C$  = (gender, diabetes, height, acute myocardial infarction within 7 days before pci, no. of vessels treated, stent deployment, lower left ventricular ejection fraction)

Patients receiving **augmented PCI** **more severely** diseased than **PCI** patients





# Results using different methods

Method	Treated	Not-treated	Risk difference (95%CI)
No adjustment	698	298	-0.0346(-0.061, -0.008)
Adjusted for Diabetes & Vessels			-0.0319 (-0.057, -0.006)
Matching on PS	Actual matches on PS		Risk difference (SE)
Nearest neighbour matching	698	247	-0,067 (0,022)
Radius matching	698	297	-0,032 (0,014)
Regression on PS	698	298	-0,034 (0,0131)

# Limitations of Propensity score

- Uncertainty due to estimating ps & stratification/matching on ps may not be taken account by standard statistical software
- Matching algorithms? 1:1 matching? 1:N matching? how close are 2 PS?
- Incomplete matchings
- Unmeasured confounding
- Which confounders in PS model

# **Summary**

- **PS conditional probability of treatment given confounders**
- **PS summarizes confounders**
- **PS use in stratification, matching, as a confounder**
- **Individuals with same propensity score equally likely to have been treated thus comparable**

# References

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2. **Rosenbaum, P.R. & Rubin, D.B. (1984)**, "Reducing bias in observational studies using subclassification on the propensity score", *Journal of the American Statistical Association*, 79, 516-524
3. **Robins J.M. Mark S.D. & Newey W.K. (1992)**, "Estimating exposure effects by modelling the expectation of exposure conditional on confounders", *Biometrics* 48, 479-495
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5. **Faries, D. Obenchain R.L. Haro J.M. & Leon A.C. (2010)**, "Analysis of observational health care data using SAS® ", Cary, NC: SAS Institute
6. **Lanehart Rheta E. et al (2012)**, "Propensity score analysis and assesmnet of propensity score approaches using SAS® procedures", paper 314-2012, SAS Global forum 2012

# Questions & Comments

